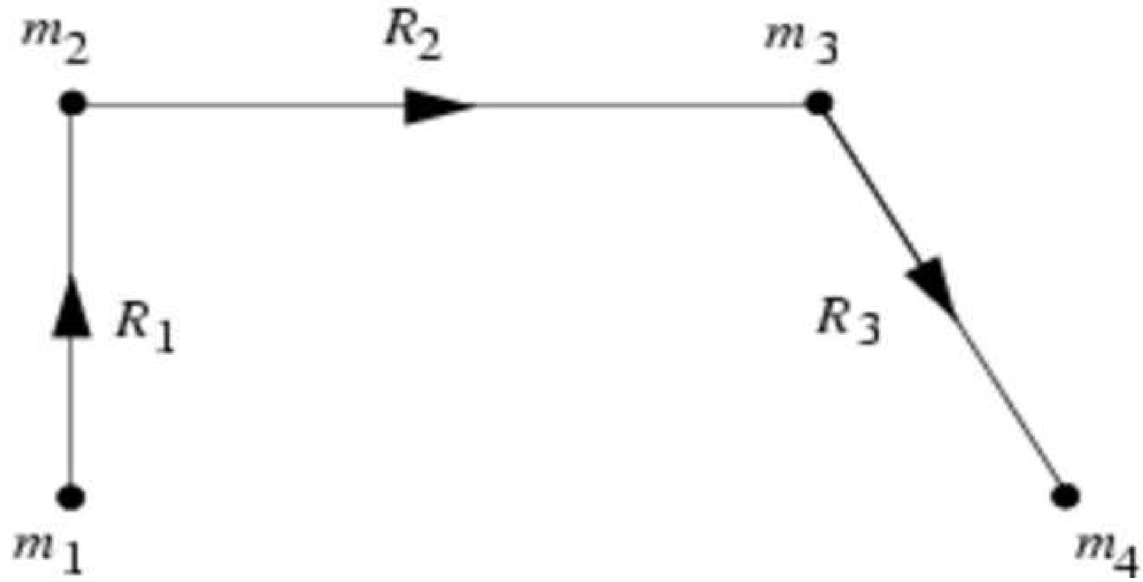
m_2
R_2
m_3
R_1
R_3
m_1
m_4

Chain Regularization

Chain vectors $\mathbf{R}_k = \mathbf{r}_{k+1} - \mathbf{r}_k; \quad k = 1, ..., N-1$

Physical momenta $\mathbf{p}_k = m_k \mathbf{v}_k; \quad k = 1, ..., N$

Relative momenta $\mathbf{W}_k = \mathbf{W}_{k-1} - \mathbf{p}_k; \quad k = 2, ..., N-2$

Hamiltonian

$$H \;=\; \frac{1}{2}\sum_{k=1}^{N-1}\left(\frac{1}{m_k} + \frac{1}{m_{k+1}}\right)\mathbf{W}_k^2 - \sum_{k=2}^{N-1}\frac{1}{m_k}\mathbf{W}_{k-1}\cdot\mathbf{W}_k - \\ - \sum_{k=1}^{N-1}\frac{m_k m_{k+1}}{R_k} - \sum_{1\le i\le j-2}^{N}\frac{m_i m_j}{R_{ij}}$$

Equations of motion

$$\frac{d\mathbf{Q}_k}{d\tau} = \frac{\partial\Gamma^*}{\partial\mathbf{P}_k}; \qquad \frac{d\mathbf{P}_k}{d\tau} = -\frac{\partial\Gamma^*}{\partial\mathbf{Q}_k}$$

KS relations $\mathbf{R}_k = \mathcal{L}_k\,\mathbf{Q}_k; \quad \mathbf{W}_k = \mathcal{L}_k\,\mathbf{P}_k/2\mathbf{Q}_k^2$

Time transformation $dt = g d\tau\,; \quad g = 1/L$

Regularized Hamiltonian $\Gamma^* \;=\; g(H - E)$

Regular solutions $R_k \to 0; \qquad k = 1, ..., N-1$

Chain Decision-Making

Perturber search	$R = a(1+e), \quad \Delta t_{\rm cm} < \Delta t_{\rm cl}$
Selection criterion	$d_j < R_{\rm cl}, \ \dot{d}_j < 0, \ j \le N$ or $j > N$
Binary termination	$\mathrm{KS} \Rightarrow \mathrm{S} + \mathrm{S}, \quad t = t_{\rm block}$
Chain initialization	$\mathbf{Q}, \mathbf{P}$ from $m_i, \mathbf{r}_i, \dot{\mathbf{r}}_i$
Time inversion	$\Delta\tau = \int L\, dt, \ \Delta t = t_{\max} - t$
Slow-down procedure	$\gamma = \dfrac{8a^3}{m_{\rm b}} \sum \dfrac{m_j}{r_{ij}^3}, \quad \kappa = \left(\dfrac{\gamma_0}{\gamma}\right)^{1/2}$
Collision test	$\min\{R_k\} < f \max(r_k^*, r_{k+1}^*)$
Addition of member	$\sum R_k + d_j < R_{\rm cl}, \ \dot{d}_j < 0$
Escape	$\frac{1}{2}\dot{d}^2 - M/d > 0, \ d > R_{\rm cl}, \ \dot{d} > 0$
Termination	$\max\{R_j\} > R_{\rm cl}, \ \dot{R}_k > 0$
Stability check	$\mathrm{B-B} \Rightarrow \mathrm{T} + \mathrm{S}, \quad a_{\rm out}(1-e_{\rm out}) > \Psi a_{\rm in}$
Time quantization	$t_{\rm new} = t_{\rm prev} + [(t - t_{\rm block})/\delta t]\, \delta t$
Re-initialization	$R_{ij}^2 \Rightarrow \mathrm{KS} + \mathrm{S} + \mathrm{S}$, or $\mathrm{KS} + \mathrm{KS} + \mathrm{S}$

Chain Procedures

Initialize in c.m. frame	$\sum m_i \mathbf{r}_i = 0, \quad \sum m_i \dot{\mathbf{r}}_i = 0$
Total energy of subsystem	$E = \frac{1}{2} \sum m_i \mathbf{v}_i^2 - \sum \dfrac{m_i m_j}{r_{ij}}$
Select chain indices & vectors	$\mathbf{Q}, \mathbf{P}, \quad N_{\rm eq} = 8\,(N-1)$
Define useful quantities	$T_{\rm cr},\ R_{\rm grav},\ \Delta\tau_0$
Form perturber list	$d < \left(\dfrac{2m}{M_{\rm ch}\gamma_0}\right)^{1/3} R_{\rm grav}$
Check time-step	$\Delta\tau = \int L\,dt, \quad L = T - \Phi$
B-S integration step	$\mathbf{r}_i = ((\frac{1}{6}\dot{\mathbf{F}}_i \delta t_i + \frac{1}{2}\mathbf{F}_i)\delta t_i + \dot{\mathbf{r}}_i)\delta t_i$
Transform to physical variables	$\mathbf{R}_k = \mathcal{L}\,\mathbf{Q}_k, \quad \mathbf{W}_k = \dfrac{\mathcal{L}\,\mathbf{P}_k}{2\,\mathbf{Q}_k^2}$
Check slow-down & switching	$\gamma < \gamma_0, \quad R_{12} < \max(R_1, R_2)$
Termination test	$\dot{R}^2 > 2M/R, \quad R > R_{\rm cl}$
Chain as decision tool	$a \ldots b \ldots\ldots\ldots\ldots\ldots\ldots c \ldots d$
Continue N-body integration	$t > t_{\max} = t_{\rm blk}$