The long-term evolution and initial size of comets 46P/Wirtanen and 67P/Churyumov-Gerasimenko

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ABSTRACT
We present a new method to study the long term evolution of cometary nuclei in order to retrieve their original size. We considered the two cases of comet 46P/Wirtanen and 67P/Churyumov-Gerasimenko. We calculated the past evolution of the orbital elements of both comets over 100,000 years using a Bulirsch-Stoer integrator and over 450,000 years using a Radau integrator, that we combined with a realistic model for the erosion of the nucleus to estimate their original size. We derived that for the long term evolution of the orbital elements of a comet, the main effect is the chaos resulting from close encounters with planets, and this effect is dominant whatever the choice of the integrator or the presence or not of non-gravitational forces. We estimated the dynamical lifetime of comet 46P to \(133,000\) years and \(105,000\) years for comet 67P. For the erosion, our model assumes a spherical nucleus made of a macroscopic mixture of pure dust and pure water ice, thermally decoupled. Erosion depends strongly on the active fraction and density of the nucleus. Assuming a density of \(300\,\text{kg}\,\text{m}^{-3}\) and an average active fraction over time of \(10\%\), we obtain an initial radius of \(1.3\,\text{km}\) for 46P and \(2.8\,\text{km}\) for 67P. The upper limit is obtained assuming a density of \(100\,\text{kg}\,\text{m}^{-3}\) and an active fraction of \(100\%\), and corresponds to an initial radius \(<20.6\,\text{km}\) for 46P and \(<25.3\,\text{km}\) for 67P. Erosion acts like a rejuvenating process of the surface so that exposed materials on the surface may not contain, or only very little, primordial materials, but materials located just under it (a few centimeters to meters) may still be very less evolved. Erosion due to water only occurs under \(4\,\text{AU}\) and is restricted to short periods, \(\sim7\%\) of the lifetime of a comet. We will apply this method to several other comets in the future.

Key words: comets: general – comets: individual: 46P/Wirtanen, 67P/Churyumov-Gerasimenko

1 INTRODUCTION
The study of comets is of paramount importance to understand and constrain the current scenario for the formation and evolution of the Solar System. A key question is to know whether comets are primordial bodies from the early formation of the Solar System, as proposed by Safronov (1991), or whether they are fragments of more recent collisions between Tran-Neptunian Objects (TNOs), as proposed by Stern (1995) and Farinella & Davis (1996). The collisional scenario can be tested, in principle, as it implies a slope of \(-2.5\) for the cumulative initial size distribution function of comets, assuming a collisionally relaxed population for the TNOs (Dohnanyi 1969). The recent progress in observation of comets allowed to determine the size of the nucleus of more than sixty ecliptic comets (Lamy et al. 2004), and it is now tempting to try to estimate the slope of their initial size distribution function, taking into account their erosion and the past evolution of their orbital elements, and check its agreement with the collisional scenario.

The study of the long-term evolution of comets suffers several problems such as the chaoticity of the orbit due to close encounters with planets and frequent occurrence of temporary residence in resonance with the mean motion of Jupiter, or the presence of non-gravitational forces (hereafter NGF). However, several numerical codes have been successfully developed in the past to tackle this problem, most
of them based on the Opik-Arnold approach (Opik 1951; Arnold 1965). We propose here a new method to estimate the initial size of cometary nuclei, which combines a numerical code for the long-term evolution of the orbital elements of a comet with a model for the erosion of its nucleus. We apply this new method to two comets, targets of space missions, 46P/Wirtanen (hereafter 46P) and 67P/Churyumov-Gerasimenko (hereafter 67P).

46P is the former target of the ESA’s Rosetta mission and has been selected as a possible target for the Comet Surface Sample Return mission which is under consideration in the framework of the NASA Discovery program. 67P is the target of the Rosetta mission with a rendez-vous on 2014.

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In a first approximation, it is possible to make a rough estimation of the initial size of a comet nucleus. Indeed, the mean lifetime of an ecliptic comet is 450,000 years (Levison & Duncan 1994), and Groussin & Lamy (2003) estimated the erosion of the nucleus of 46P, a typical ecliptic comet, to ~0.5 m per orbit, or ~0.1 m per year. So, the resulting erosion and hence the initial size is 450,000×0.1=45 km. However, as we will see in this paper, this is a rough estimate and the more realistic calculations of the initial size presented in this paper lead to different conclusions.

In section 2 we present the method we used to calculate the long-term evolution of the orbital elements of 46P and 67P. In section 3 we present our model for the erosion of the nucleus. Results and discussions are presented in section 4 for the long-term evolution of the orbital elements and in section 5 for the erosion of the nucleus and the estimation of its initial size. Conclusions are given in section 6.

2 TWO NUMERICAL CODES TO STUDY THE LONG-TERM EVOLUTION OF THE ORBITAL ELEMENTS

We used two different numerical codes to study the long-term evolution of small bodies in the Solar System. Each code takes into account (or not) the NGF and includes the gravitational influence of all the planets except Pluto, whose mass in negligible, and Mercury, whose mass is added to that of the Sun. The values used for the NGF parameters are $A_1=6.0\times10^{-9}\text{AU}\text{day}^{-2}$ and $A_2=-1.0\times10^{-9}\text{AU}\text{day}^{-2}$ for 46P (Horizon 2000 on 1997) and $A_1=0.7\times10^{-9}\text{AU}\text{day}^{-2}$ and $A_2=0.1\times10^{-9}\text{AU}\text{day}^{-2}$ for 67P (MPC 34423).

Table 2. Initial radius and mass of the 8 planets considered in our analysis plus 46P and 67P.

<table>
<thead>
<tr>
<th>Body</th>
<th>Radius (km)</th>
<th>Mass (M_{\oplus})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun+Mercury</td>
<td>800000</td>
<td>1.00000016601368</td>
</tr>
<tr>
<td>Venus</td>
<td>6070</td>
<td>0.2447695979669\times10^{-5}</td>
</tr>
<tr>
<td>Earth+Moon</td>
<td>6378</td>
<td>0.3040427387108\times10^{-5}</td>
</tr>
<tr>
<td>Mars</td>
<td>3395</td>
<td>0.3227143621539\times10^{-6}</td>
</tr>
<tr>
<td>Jupiter</td>
<td>71300</td>
<td>0.95479066214732\times10^{-3}</td>
</tr>
<tr>
<td>Saturn</td>
<td>60100</td>
<td>0.2858776436821\times10^{-3}</td>
</tr>
<tr>
<td>Uranus</td>
<td>26150</td>
<td>0.435540668641\times10^{-4}</td>
</tr>
<tr>
<td>Neptune</td>
<td>19332</td>
<td>0.51775913844879\times10^{-4}</td>
</tr>
<tr>
<td>46P</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>67P</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The first code, implemented by R. Gonczi, uses a Bulirsch-Stoer (Stoer & Bulirsch 1980) step-size algorithm, which is optimized for dealing with close encounters: with this kind of algorithm, the time step is automatically reduced during the encounters, in a way which is not determined “a priori” but depends on the geometry and speed of the encounters. The second code, implemented by G. Hahn, is a 15th order Radua integrator (Everhart 1985), which allows variable size of the integration step, internally adjusted to keep a chosen accuracy (we used $10^{-8}$) during close encounters. The force routine of the Radua integrator was modified to take into account NGF parameters (Marsden 1970), the effect of which were evaluated at each integration step.

The chaotic behavior is simulated by considering 9 different values of the initial mean anomaly separated by 0.00001°, from 359.79226° to 359.79234° for 46P and from 164.68808° to 164.68616° for 67P. Each initial mean anomaly represents what is usually called a clone (Dones et al. 1996). As we performed the simulation with and without NGF, we have 18 clones for each comet; 9 different mean anomaly times 2 (with/without NGF). The initial orbital elements of the clones we used for the simulation are given in Table 1 and the mass and radius of the bodies in Table 2. All orbital elements refers to 1997 March 14.

We integrated backward over 100,000 years with the Bulirsch-Stoer integrator, and over 450,000 years with the Radua integrator. The first integration time, 100,000 years, is sufficient to compare the two different methods (Bulirsch-Stoer v.s. Radua) and to compare the cases with and without NGF. The second integration time, 450,000 years, is the mean lifetime for ecliptic comets determined by Levison & Duncan (1994). For each clone, we stoped the simulation when it escapes the inner Solar System, that is $a>100\text{AU}$ or $e>1$ (hyperbolic orbit). This choice implies that we neglect possible multiple injections of a given clone in the inner Solar System; to avoid this problem, the only solution would be to integrate over the entire age of the Solar System, that is $\sim4.5\times10^9\text{years}$, but it is currently not feasible for computer-time reasons.
The model for the erosion of the nucleus must remain valid over 450,000 years, which has strong implications. We assume a spherical nucleus. This assumption is justified for 46P and 67P since both comets are almost spherical, with an elongation of ~1.2 for 46P (Lamy et al. 1998) and ~1.3 for 67P (Lamy et al. 2006). We assume a nucleus made of a macroscopic mixture of pure dust and pure water ice, thermally decoupled, i.e., one temperature for the pure dust regions and one for the pure water ice regions and one for the pure dust regions. This assumption is justified by the Deep Impact observations of the nucleus of comet 9P/Tempel 1 (Sunshine et al. 2005; Groussin et al. 2006), which indicate that water ice and dust are thermally and then physically decoupled at a macroscopic scale. The fraction of the nucleus that is active is a free parameter since it varies from one comet to the other and it can also change with time. Recent works indicate an active fraction of ~100% at perihelion and ~10% at aphelion for 46P (Groussin & Lamy 2003), and ~10% at perihelion for 67P (Lamy et al. 2006). We assume that active regions are numerous and distributed all over the nucleus. Since localized active regions will create cracks or weel as they erode, we also assume that the position of active regions change over long timescale, as well as the pole orientation, so that the erosion remains roughly uniform, an argument already used by Hughes (2003). Heat conduction is not taken into account. This assumption is justified by several observations that indicates a very low thermal inertia on comets and Centaurs (e.g., Fernández et al. 2002; Groussin et al. 2006). Finally, species more volatiles than H$_2$O, such as CO or CO$_2$, are not taken in to account, as they are not expected to be directly on the surface, and consequently only play a minor role on the erosion compared to H$_2$O.

The surface energetic balance of our model is given by:

$$\frac{(1-A)F_{\text{sun}} \cos z}{r^2} = \eta \sigma T^4 + f(1 - \alpha_R) L(T) Z(T)$$

(1)

where $A = p e q$ is the product of the geometric albedo $p_e$ by the phase integral $q$, $F_{\text{sun}}$ [Wm$^{-2}$] is the solar constant, $r$ [AU] is the heliocentric distance, $z$ is the zenithal angle, $\eta$ is the beaming factor introduced by Lebofsky et al. (1986), $\sigma$ [JK$^{-1}$m$^{-2}$s$^{-1}$] is the Stefan Boltmann’s constant, $T$ [K] is the surface temperature, $f$ is the fraction of water ice ($f=0$ for pure dust or $f=1$ for pure water ice), $\alpha_R$ accounts for the recondensation of water ice on the surface (Crifo 1987), $L$ [Jkg$^{-1}$] is the latent heat of sublimation of water ice, $Z(T)$ [kg s$^{-1}$m$^{-2}$] is the H$_2$O sublimation rate. For the latent heat of sublimation of water ice $L$, we took a standard value:

$$L = 2.8 \times 10^6 \text{Jkg}^{-1}$$

(2)

The H$_2$O sublimation rate $Z(T)$ is given by:

$$Z(T) = P_v(T) \sqrt{\frac{M}{2 \pi R T}}$$

(3)

where $M=18$ g mol$^{-1}$ is the H$_2$O molecular weight and $R=8.314$ JK$^{-1}$mol$^{-1}$ is the gas constant. The vapor pressure $P_v(T)$ [Pa] is given by Fanale & Salvail (1984) and Skorov et al. (1999):

$$P_v(T) = A \exp \left( -\frac{B}{T} \right)$$

(4)

where $A=3.56 \times 10^{12}$ Pa and $B=6162$ K. The numerical values of the other parameters will be discussed in section 3.2.

The water production rate [kg s$^{-1}$] is calculated for each point of the surface and then integrated:

$$Q_{H2O} = 2 \pi r_a^2 \int Z(T(z, r, f = 1)) \sin zdz$$

(5)

where $r_a$ [m] is the nucleus radius and $z$ is the active fraction $0 \leq z \leq 1$ with $x = 1$ for a 100% active nucleus.

The resulting global erosion [m s$^{-1}$] is given by:

$$E = \frac{Q_{H2O}}{4 \pi r_a \rho}$$

(6)

where $\rho$ [kg m$^{-3}$] is the bulk density of the nucleus. That is, using Eq. (5):

$$E = \frac{\int Z(T(r, z, f = 1)) \sin zdz}{2 \rho}$$

(7)
For each clone, we use the following method for the integration as a function of time $t$:

1) we take the orbital elements $a$ and $e$ at perihelion, provided by either the Bulirsch-Stoer or Radau integrator, and we derive the revolution period $P$ using Kepler’s equations. If $0 < a < 100$ and $0 < e < 1$ we continue, otherwise we stop the integration for this clone and restart with another clone.

2) we integrate from $t = 0$ to $t = P$, that is one revolution period, using a variable time step $dt$ as function of heliocentric distance: 1 hour under 0.2 AU, 1 day under 1 AU, 5 days under 2 AU, 10 days under 5 AU, 30 days under 20 AU, 1 year under 50 AU and 10 year beyond. On each time interval $[t, t + dt]$ we calculate: (i) the heliocentric distance using the Kelper’s equations, (ii) the surface temperature distribution as a function of heliocentric distance using Eq. (1), (iii) the water production rate using Eq. (5), and (iv) the erosion using Eq. (6). The net erosion over the interval $[t, t + dt]$ is $\Delta E = \int_{t}^{t + dt} \dot{E} \, dt$.

We repeat (2) until the clone reaches perihelion again. Then, we go to (1) and take the orbital elements at the next perihelion passage. Time steps are short enough to insure convergence on the erosion. As our model does not consider heat conduction, temperature (and erosion) is nil on the night side.

Eq. (7) shows that erosion does not depend on the size of the considered body as the term $r_{p}^{2}$ vanishes between Eq. (5) and Eq. (6). It means that large nuclei have the same erosion rate $[\text{m s}^{-1}]$ than small nuclei. Moreover, Eq. (7) shows that erosion depends on the temperature distribution $T(r, z)$ and only two parameters, $x$ and $\rho$. The lower the active fraction $x$, the lower the erosion rate since less water ice sublimates. The lower the density $\rho$, the higher the erosion, as the same amount of materials uses a larger space.

In spite of the above limitations, this model is more realistic than previous models as it calculates the erosion as a function of heliocentric distance with a variable time step and takes into account the temperature variations on the surface, while previous models only used average erosion (Weissman & Lowry 2002) or uniform temperature and constant perihelion distance (Hughes 2003).

### 3.2 The parameters of the model

The various parameters involved in our model are not known for cometary nuclei. We discuss below how we selected their respective values.

The infrared emissivity $\epsilon$ is taken equal to 0.95, the middle point of the interval 0.9-1.0 always quoted in the literature. As the interval is very small and the value near 1.0, this uncertainty has a negligible influence on the calculated surface temperature and consequently, on the water production rate.

The beaming factor $\eta$ reflects the influence of surface roughness which produces an anisotropic thermal emission. The values of $\eta$ determined for a few asteroids and satellites vary from 0.7 to 1.2 (Spencer et al. 1989; Harris 1998). The value of $\eta=0.756$, derived from observations of 1 Ceres and 2 Pallas by Lebofsky et al. (1986) has often been considered a standard and used for other solar system objects (e.g., Centaur 1997 CU26, Jewitt & Kalas 1998). However, for low albedo objects such as cometary nuclei, Lagerros (1998) pointed out that a rather high surface roughness is required in order to achieve this value. His recommendation led us to select the more appropriate value $\eta=0.85$. This value is also in agreement with the in-situ measurements of the surface temperature of comet 9P/Tempel 1 by Deep Impact (Groussin et al. 2006). As the temperature varies as $\eta^{-1/4}$, the beaming factor has an important effect on the water production rate and then erosion, especially at large heliocentric distances: $\leq 10\%$ for $r \leq 2$ AU and $\eta$ in the range 0.756-0.85, but $\geq 60\%$ for $r > 4$ AU.

The Bond albedo $A = p_{\nu} q$ requires a knowledge of the geometric albedo $p_{\nu}$ and of the phase integral $q$ which measures the angular dependence of the scattered radiation. We chose $p_{\nu}=0.04$, which is a canonical value used for cometary nuclei, and $q=0.27$, the value derived by Buratti et al. (2004) from the Deep Space 1 observations of 19P/Borrelly. This choice is reinforced by the value of $q=0.28$ found for 253 Mathilde (Clark et al. 1999), since the surface properties of this asteroid ($p_{\nu}=0.047$ and $q=0.04$ mag deg$^{-1}$) are typical of cometary nuclei.

The recondensation of water ice on the surface is discussed in details in Crifo (1987). We adopted his recommended value $\alpha_{R}=0.25$.

Even if we have estimates of the current value of $x$ for 46P and 67P, we selected a range of values for $x$ since active fraction may vary with time. We use $x = 0.03, 0.10, 0.50$ and 1.

The value of $\rho$ is unknown and we use $\rho = 100, 300$ and 500 kg m$^{-3}$. This range comes from the bulk density derived for Tempel 1 with the Deep Impact experiment, that is $300\pm200$ kg m$^{-3}$ (Richardson & Melosh 2006).

One can easily derive the erosion for other values of $x$ and $\rho$ from Tables 3, using Eq. (7) and a rule of three.

### 4 RESULTS AND DISCUSSION ON DYNAMIC: LONG-TERM EVOLUTION OF THE NUCLEUS

We performed the backward integration of the orbital elements of comet 46P using the Bulirsch-Stoer and Radau integrators, and that of comet 67P using the Radau integrator only, so that we have three different dataset. For each dataset, we have 9 different values for the initial mean anomaly, with or without NGF, that is 18 different clones. We integrated each clone until it escapes the inner Solar System ($a > 100$ AU or hyperbolic orbit), with a maximum integration time of 100,000 years for Bulirsch-Stoer and 450,000 years for Radau. Table 3 summarizes the results.

For 46P with Bulirsch-Stoer, 1 clone crashes on Jupiter, 9 clones are ejected from the inner Solar System and 8 clones are still in the inner Solar System after 100,000 years. For 46P with Radau, 15 clones are ejected from the inner Solar System and 3 are still in the inner Solar System after 450,000 years. For 67P with Radau, 14 clones are ejected from the inner Solar System and 4 are still in the inner Solar System after 450,000 years.

The Bulirsch-Stoer and Radau methods give comparable results for the long-term evolution of the orbital parameters, as illustrated by Fig. 1. It illustrates the evolution of the perihelion distance $q = a(1 - e)$ as a function of time, for the different clones of 46P without NGF, and
The long-term evolution and initial size of comets 46P and 67P

Table 3. Summary of the evolution (lifetime and endstate) and erosion (in km, for different values of active fraction $x$) of the orbits, depending on the initial mean anomaly $M$ and on the presence or not of Non Gravitational Forces (NGF).

<table>
<thead>
<tr>
<th>Mean anomaly</th>
<th>NGF</th>
<th>Lifetime (yr)</th>
<th>Endstate</th>
<th>$x=1.00$</th>
<th>$x=0.50$</th>
<th>$x=0.10$</th>
<th>$x=0.03$</th>
</tr>
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<tbody>
<tr>
<td>$M_{-4}=359.79226^o$</td>
<td>No</td>
<td>51923 a&gt;100</td>
<td>4.28</td>
<td>2.14</td>
<td>0.43</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>84274 a&gt;100</td>
<td>2.00</td>
<td>1.00</td>
<td>0.20</td>
<td>0.06</td>
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</tr>
<tr>
<td>$M_{-3}=359.79227^o$</td>
<td>No</td>
<td>11920 Hyperbole</td>
<td>0.65</td>
<td>0.3</td>
<td>0.07</td>
<td>0.02</td>
<td></td>
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<tr>
<td></td>
<td>Yes</td>
<td>45991 a&gt;100</td>
<td>10.19</td>
<td>5.10</td>
<td>1.02</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>$M_{-2}=359.79228^o$</td>
<td>No</td>
<td>&gt;100000</td>
<td>5.56</td>
<td>2.78</td>
<td>0.56</td>
<td>0.17</td>
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<td></td>
<td>Yes</td>
<td>48515 a&gt;100</td>
<td>4.56</td>
<td>2.28</td>
<td>0.46</td>
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<tr>
<td>$M_{-1}=359.79229^o$</td>
<td>No</td>
<td>&gt;100000</td>
<td>3.95</td>
<td>1.98</td>
<td>0.40</td>
<td>0.12</td>
<td></td>
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<tr>
<td></td>
<td>Yes</td>
<td>34959 Hyperbole</td>
<td>13.32</td>
<td>6.66</td>
<td>1.33</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>$M_0=359.79230^o$</td>
<td>No</td>
<td>24458 Hyperbole</td>
<td>5.73</td>
<td>2.87</td>
<td>0.5</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>3492 Collision Jupiter</td>
<td>3.03</td>
<td>1.52</td>
<td>0.30</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>$M_{+1}=359.79231^o$</td>
<td>No</td>
<td>&gt;100000</td>
<td>25.73</td>
<td>12.86</td>
<td>2.57</td>
<td>0.78</td>
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</tr>
<tr>
<td></td>
<td>Yes</td>
<td>&gt;100000</td>
<td>19.62</td>
<td>9.81</td>
<td>1.96</td>
<td>0.59</td>
<td></td>
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<tr>
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<td>&gt;100000</td>
<td>3.70</td>
<td>1.85</td>
<td>0.37</td>
<td>0.11</td>
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<tr>
<td></td>
<td>Yes</td>
<td>&gt;100000</td>
<td>8.74</td>
<td>4.37</td>
<td>0.87</td>
<td>0.26</td>
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<tr>
<td>$M_{+3}=359.79233^o$</td>
<td>No</td>
<td>&gt;100000</td>
<td>8.03</td>
<td>4.02</td>
<td>0.8</td>
<td>0.24</td>
<td></td>
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<tr>
<td></td>
<td>Yes</td>
<td>&gt;100000</td>
<td>1.80</td>
<td>0.90</td>
<td>0.18</td>
<td>0.05</td>
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<tr>
<td>$M_{+4}=359.79234^o$</td>
<td>No</td>
<td>27185 a&gt;100</td>
<td>1.44</td>
<td>0.72</td>
<td>0.14</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>89404 Hyperbole</td>
<td>7.70</td>
<td>3.86</td>
<td>0.77</td>
<td>0.23</td>
<td></td>
</tr>
</tbody>
</table>

Median value for different densities

- $\rho=500$ kg m$^{-3}$: 3.34, 1.67, 0.33, 0.10
- $\rho=300$ kg m$^{-3}$: 5.56, 2.78, 0.56, 0.17
- $\rho=100$ kg m$^{-3}$: 16.68, 8.34, 1.67, 0.51

for the Bulirsch-Stoer and Radau method. The perihelion distance is the most appropriated parameter to study the erosion, but we obtained similar conclusions for the other orbital parameters $i$, $\omega$ and $\Omega$. The evolution of the orbital elements is strongly connected to the close approaches to Jupiter, and each close approach induces some divergence between the orbital elements of the different clones. The Bulirsch-Stoer and Radau method use different time steps and the resulting orbits for the different clones are not strictly identical. Even if they only give comparable values of $q$ during the first hundred years, the main point for this paper is that after 1,000 years, the orbits are already chaotic, and that after 100,000 years, the perihelion distance can be whatever value between 0 and >10 AU. So, for the long-term integration, chaoticity is the dominant effect, with non-deterministic orbits, and we cannot favor one method to the other (Bulirsch-Stoer or Radau).

Fig. 2 shows the influence of NGF. It illustrates the evolution of the perihelion distance $q = a(1 − e)$ as a function of time, for the different clones of 67P, with and without NGF. Until the first close approach to Jupiter, the influence of NGF is negligible. In 1959, we retrieve the close approach at 0.052 AU from Jupiter already mentioned by Krolikowska (2003), in all our runs (with or without NGF).
from Burslich-Stoer and Radau it the combination of: (i) a
the time it takes to eject half of the clones out of the in-
Stoer, Radau. The median lifetime corresponds to
perbolic orbit or
for 67P with Radau. The median lifetime corresponds to
mation since at that distance it is less than 1% of the water
comparable to the lifetime itself, and (ii) a low statistics of
the clones with NGF or without NFG.

This close approach makes the long-term evolution before it
completely non-deterministic and on the long-term, chaotic-
ity is the dominant effect. After 2,000 years, and even more
after 100,000 years, orbits become non-deterministic and the
periheilon distance can be whatever value between 0 and
>10 AU. So, for the purpose of this paper which is to esti-
mate the erosion over several hundred thousand years, there
is no reason to favor any NGF parameters, and we will use
equally the clones with NGF or without NFG.

The time at which a clone escape the Solar System (hy-
perbolic orbit or \(a > 100\) AU) is its dynamical lifetime, and
it corresponds to the time since its last injection in the in-
ner Solar System. We obtain a median dynamical lifetime of
\(~89,000\) years for the different clones of 46P with Bulirsch-
Stoer, \(~133,000\) year for 46P with Radau and 105,000 year
for 67P with Radau. The median lifetime corresponds to
the time it takes to eject half of the clones out of the in-
ner Solar System. We believe that the discrepancy of a fac-
tor \(~1.5\) between the dynamical lifetime of 46P estimated
from Burslsch-Stoer and Radau it the combination of: (i) a
too short integration time for Bulirsch-Stoer (100,000 years),
comparable to the lifetime itself, and (ii) a low statistics of
only 18 clones. For this reason, we favor the results of Radau
with a longer integration time, i.e., a dynamical lifetime of
\(~133,000\) year for 46P and \(~105,000\) year for 67P. This time
is shorter than the typical median dynamical lifetime of
450,000 years calculated by Levison & Duncan (1994) for
short-period comets.

Finally, we estimated the percentage of time the comet
is active and inactive, define by the ratio of the time the
comet is below 4 AU over the time it is further then 4 AU.
We consider 4 AU as a reasonable limit for water ice subli-
mation since at that distance it is less than 1% of the water
production rate at 1 AU. We obtained a median value for
the different clones of 7% for 46P with Bulirsch-Stoer, 5% for
46P with Radau, and 7% for 67P with Radau. Levison
& Duncan (1994) estimated that comets passes 7% of their
time under 2.5 AU, which is in good agreement with our re-
results. The main conclusion is that comets are inactive more
than 90% of their time, and that they only experience ero-
sion on a very short time compared to their total dynamical
lifetime.

5 RESULTS AND DISCUSSION ON EROSION:
INITIAL SIZE OF THE NUCLEUS
We used the model for the erosion of the nucleus presented in
section 3 to calculate the erosion for each clone of Table 3 in
order to retrieve their initial size. This table summarizes the
results for different density from 100 kg m\(^{-3}\) to 500 kg m\(^{-3}\)
and different active fraction from 3% to 100%. As we have
no reason to favor one evolution scenario to the other, we
calculated the median value of the erosion for all the clones.

Fig. 3 and 4 illustrates the erosion of the different clones
for 46P with Bulirsch-Stoer and Radau respectively, assuming
a density of 300 kg m\(^{-3}\) and an active fraction of 100%. Erosion is strongly connected to the evolution of the perihe-
lion distance, so that each clone has its own erosion history. Clones that escape the inner Solar System do not reach the
maximum integration time. Erosion increases with time, as
expected. However, the erosion rate is far from being con-
tant, and each clone alternates between period of strong,
fast erosion and period of low or no erosion. This is repre-
sented by the almost step function of erosion as a function
of time. This is also in agreement with the result of section

<table>
<thead>
<tr>
<th>Mean anomaly</th>
<th>NGF</th>
<th>Lifetime (yr)</th>
<th>Endstate</th>
<th>(x=1.00)</th>
<th>(x=0.50)</th>
<th>(x=0.10)</th>
<th>(x=0.03)</th>
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<tr>
<td>(67P/Churyumov-Gerasimenko - Radau method) - (\rho=300) kg m(^{-3})</td>
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<td></td>
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<tr>
<td>(M_{-4}=164.68608^\circ)</td>
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<td>&gt;500000</td>
<td>1.35</td>
<td>0.68</td>
<td>0.14</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
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<td>3.24</td>
<td>1.62</td>
<td>0.32</td>
<td>0.10</td>
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<tr>
<td>(M_{-3}=164.68609^\circ)</td>
<td>No</td>
<td>19945 (a&gt;100)</td>
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<td>0.92</td>
<td>0.18</td>
<td>0.05</td>
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<tr>
<td>&amp; Yes</td>
<td>73489 (a&gt;100)</td>
<td>0.51</td>
<td>0.26</td>
<td>0.05</td>
<td>0.02</td>
<td></td>
<td></td>
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<tr>
<td>(M_{-2}=164.68610^\circ)</td>
<td>No</td>
<td>105446 (a&gt;100)</td>
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<td>3.48</td>
<td>0.70</td>
<td>0.21</td>
<td></td>
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<tr>
<td>&amp; Yes</td>
<td>62281 Hyperbole</td>
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<td>1.59</td>
<td>0.32</td>
<td>0.10</td>
<td></td>
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<tr>
<td>(M_{-1}=164.68611^\circ)</td>
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<td>113737 (a&gt;100)</td>
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<td>10.70</td>
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<tr>
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<td>1.70</td>
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<tr>
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<td>No</td>
<td>238832 (a&gt;100)</td>
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<tr>
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<td>177376 (a&gt;100)</td>
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<tr>
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<td>&gt;500000</td>
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<tr>
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<td>58553 (a&gt;100)</td>
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<tr>
<td>&amp; Yes</td>
<td>63557 (a&gt;100)</td>
<td>18.57</td>
<td>9.29</td>
<td>1.86</td>
<td>0.56</td>
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</tbody>
</table>

Median value for different densities:

\(\rho=500\) kg m\(^{-3}\): 4.67 2.33 0.47 0.14
\(\rho=300\) kg m\(^{-3}\): 7.78 3.89 0.78 0.23
\(\rho=100\) kg m\(^{-3}\): 23.34 11.67 2.33 0.70

This table summarizes the results of the erosion at 1 AU. We obtained a median value for
the different clones of 7% for 46P with Bulirsch-Stoer, 5% for
46P with Radau, and 7% for 67P with Radau. Levison
& Duncan (1994) estimated that comets passes 7% of their
time under 2.5 AU, which is in good agreement with our re-
results. The main conclusion is that comets are inactive more
than 90% of their time, and that they only experience ero-
sion on a very short time compared to their total dynamical
lifetime.
The long-term evolution and initial size of comets 46P and 67P

4, which shows that comets are inactive most of their time. As explained before, orbits become chaotic after a few hundred years, so that there is a large dispersion for values of the erosion for the different clones.

From Tables 3, the median erosion is ~20% larger for 46P using Bulirsch-Stoer than Radau, but we do not consider this as an issue since it mainly reflects our low statistic of only 18 clones in each case. For clarity, and also because the integration time is longer, we will only discuss the results for Radau, but the conclusions are similar for Bulirsch-Stoer. For a typical density of 300 kg m$^{-3}$ like Tempel 1 (Richardon & Melosh 2006), the median erosion varies from 6.67 km for an active fraction of 100% to 0.20 km for an active fraction of 3%. With the current knowledge of 46P, that is an active fraction of 100% at perihelion where most of the erosion happens (Groussin & Lamy 2003) and a radius of 0.6 km (Lamy et al. 1998), we obtain an initial radius of $0.60 + 6.67 = 7.27$ km. It is highly probable that the active fraction has changed with time, so that 7.27 km is an upper limit for a 100% active nucleus. For an average value of 10% for the active fraction over time, we derive an erosion of 0.67 km, that is an initial radius of $1.3$ km. This results also depends on the density. For an extremely low density of 100 kg m$^{-3}$ and a 100% active nucleus, we obtain an upper limit on the initial size of $0.60 + 20.01 = 20.61$ km
Figure 4. Erosion as a function of time for 46P with a Radau integrator. Density of 300 kg m$^{-3}$ and active fraction of 100%.

for 46P. This upper limit is clearly under the rough estimate of $\sim 45$ km presented in the introduction, which clearly demonstrates the importance of combining dynamical model and realistic erosion model to estimate the erosion over long-timescale.

The above results suggest that if 46P continues to erode at the same rate with a 100% active nucleus, it will soon disappear because of the process of erosion itself, in a few thousand years (Groussin & Lamy 2003). While some comets may become dormant, many of them are believed to physically disrupt (Levison et al. 2002). Our results indicate that erosion can be an important process to remove a large fraction of the surface, probably in a non-homogeneous way, creating topographic discontinuities that could help in their disruption. So, comets with currently large active fraction like 46P may be good candidates for breakup. It also implies that a comet can probably not survive a very long time (hundreds of rotation) with its entire surface active, and considering the dynamical lifetime of $\sim 133,000$ years for 46P, its active fraction was probably lower in the past.

Fig. 5 illustrates the erosion of the different clones for 67P with Radau, with a density of 300 kg m$^{-3}$ and an active fraction of 100%. The conclusions are similar to 46P, that is the erosion increases irregularly with time and after a few hundred years chaoticity induces a large range of values for the erosion for the different clones. For a typical density of 300 kg m$^{-3}$ like Tempel 1, the current active fraction of $\sim 10\%$ and a radius of 1.98 km (Lamy et al. 2006), we obtain an initial size of $1.98 + 0.78 = 2.76$ km. The upper limit on the initial size is given by a density of 100 kg m$^{-3}$ and a 100% active nucleus, that is $1.98 + 23.34 = 25.32$ km for 67P. This is still lower than the rough estimate of $\sim 45$ km given in the introduction.

For 46P and 67P, there is a large range of possibilities for the erosion, depending on the active fraction, the density, and of course the evolution of the orbital parameters. However, it appears that even with a completely different evolution of their orbital elements, 46P and 67P have comparable median erosion. For the moment, we cannot conclude from these two peculiar cases if it is generic property of all Jupiter family comets. Only a calculation using our method for a large number of Jupiter family comets will allow to confirm or infirm this result. We also note that 46P and 67P are strongly connected to Jupiter with a Tisserand parameter $> 2.5$, implying many close approaches to Jupiter and a very chaotic evolution of their orbital elements. For comets with smaller Tisserand parameter, we can expect a lower chaoticity of the orbit and a better accuracy on the determination of the erosion and initial radius.

Finally, it is interesting to see that the lowest median values of Tables 3 correspond to a global erosion of more than a hundred meters, for both 46P and 67P. While this erosion can be concentrated on a few regions, it seems that most of the nucleus should have been re-surfaced in the past, and that only a small fraction of their surface can still potentially be primordial.

6 CONCLUSIONS

This study represents another step in the comprehension of the long term evolution of cometary nuclei. We developed a new method to estimate the initial size of cometary nuclei, that we applied on 46P and 67P. We used two different numerical codes for the backward integration of the orbital elements, in order to not be biased by the code, and one realistic code for the erosion of the nucleus. Our main results are the following:

(1) For the long term evolution of the orbital elements of a comet, the main effect is the chaos resulting from close encounter with planets. Very similar orbital elements, separated by a difference on the mean anomaly of only $10^{-6}$ deg, are scattered over a large range of orbital elements after a few hundred years for Jupiter family comets such as 46P and 67P. This dispersion is present for the both Burslisch-Stoer and Radau integrator, either we include or not the non-gravitational forces.

(2) For reasonable values of 300 kg m$^{-3}$ for the density and 10% for the nucleus average active fraction, we derived an initial radius of $\sim 1.3$ km for 46P and $\sim 2.8$ km for 67P.
For an extreme low density of 100 kg m\(^{-3}\) and a 100% active nucleus, we obtain an upper limit on the initial size \(< 20.6 \text{ km}\) for 46P and \(<25.3 \text{ km}\) for 67P. Results for different values of density and active fraction can be found in Tables 3. This erosion is smaller than the rough estimation of \(<45 \text{ km}\) derived in the introduction using very simple model, which illustrates the importance of combining dynamical model and realistic erosion model to estimate the erosion over long-timescale.

(3) We estimate the dynamical lifetime of comet 46P to 133,000 years and 105,000 years for comet 67P. This lifetime is shorter than the median dynamical lifetime of 450,000 years calculated by Levison & Duncan (1994) for short-period comets. Since 46P and 67P are two peculiar cases, this agreement is reasonable.

(4) Erosion due to water only occurs under 4 AU and is restricted to short period, \(~7\%\) of the lifetime of a comet. Most of the time comets do not experience erosion.

(5) The main uncertainty on the above results is the unknown density and active fraction. As long as we will not have an idea of how active fraction changes with time, more complex models for the erosion should not provide better information. We expect future space mission like Rosetta to clarify this point.

More generally, our conclusions show that Jupiter family comets have been eroded by more than a hundred meters in the past and most probably entirely re-surfaced. The surface we observe today may not contain, or only very little, primordial materials, residuals of the early formation of the Solar System. But, on the opposite, the erosion acts like a rejuvenating process of the upper surface layers so that materials located just under the surface (a few centimeters to meters) may still be very less evolved. This last idea is supported by several other works, like the thermal analysis of the nucleus surface of comet 9P/Tempel 1 (Groussin et al. 2006).

Finally, each comet experiences its own history. It is difficult to derive general conclusions on a median erosion for all comets from the only two peculiar cases of 46P and 67P we studied, even if it seems to be less than a few kilometers. We must apply the same method to a larger number of comets, with different Tisserand parameters, in order to calculate the erosion and then retrieve the slope of the original size distribution function of cometary nuclei. This will be done in future works.

REFERENCES

Everhart, E. 1985, ASSL Vol. 115: IAU Colloq. 83: Dynamics of Comets: Their Origin and Evolution, 185

Fanale, F. P. & Salvai, J. R. 1984, Icarus, 60, 476
Harris, A. W. 1998, Icarus, 131, 291
Richardson, J. E., Melosh, H. .J., LPSC XXXVII conference, 1836